Johannes Boen, *Musica* (1357)

<PROHEMIUM>

Musicalis scientia, que sonorum respicit intervalla et de proportionibus gravis loquitur ad acutum, adeo mee rudi prelusit ymaginationi, quod sua captus immensa dulcedine stipites hos grossos legere compulsus sum pro scala, qua in posterum profundiori studio fortasse deditus ad altitudinem melius ascendere valeam, subtilius construenda.

In primis quidem docebo quoslibet reperire gradus si<n>gillatim inter G-ut et g-sol-re-ut, id est inter G primum et G secundum, que in dupla sese respiciunt proportione, et de hinc usque ad E-la, que clavis vicesima in manu ac ultima dicitur; ac nomina proportionum cuiuslibet clavis ad G primum, que G-ut dicitur, in numeris demonstrabo.

Secundo qualiter tonus dividitur in duo semitonia, scilicet maius et minus, et sic inequaliter, quis insuper sit excessus, quo minus a maiore vincitur, et quotiens excessus huiusmodi, qui comma dicitur, in tono contentus est, breviter pertractabo.

Tertio ad naturam decime clavis que b-fa-♭-mi dicitur, in qua semitonium maius vel semitonium minus et comma reperta sunt, et ad effectus litterarum eiusdem clavis, puta b et ♭, mirabilisimos in cottidiana praxi contingere possibiles leviter emigrabo.

Quarto et ultimo de proportionibus sonorum adinvicem, qui mixturam non respuunt, ex quibus quasi unus sonus mixtus exsurgit et placidus aures mulcens et alliciens, non quilibet ut integer aurem exacerbans, modicum enarrabo, ut hec pervilia, licet incompta sint et rudia, materiam saltem dent altius speculandi.

<PROLOGUE>

The musical science, which deals with the intervals of sounds and speaks of the proportions, low to high, has engaged my crude imagination so much that I, captivated by its immense sweetness, have been driven to gather these rails in order more subtly to build a ladder by which I, devoted to more profound study, may perhaps be better able hereafter to ascend to great heights.

Firstly, indeed, I shall teach how to find, one by one, any degrees you like between **G-ut** and **g-sol-re-ut**, that is, between the first **G** and the second **G**, which relate to each other in duple proportion, and from there up to **E-la**, which is called the twentieth and last key in the hand; and I shall demonstrate, in numbers, the names of the proportions of any key you like [in relation] to the first **G**, which is called **G-ut**.

Secondly, I shall briefly treat the division of the [whole] tone into two semitones, namely the greater and lesser, and thus unequally, and what, furthermore, is the quantity in excess by which the lesser is outdone by the greater, and how much of that quantity, which is called the comma, is contained in the tone.

Thirdly, I shall move on to the nature of the tenth key, which is called **b-fa-♭-mi**, in which the major semitone, the minor semitone, and the comma are found, and to the effects of the letters of the same key, that is, **b** and **♭**, [which effects are] most marvelous, and can be produced easily in daily practice.

Fourth and last, I shall say a little about the proportions between sounds that do not refuse to blend together, from which [proportions] arises a blended sound that is almost like one, and smooth, delighting and alluring the ears, not something grating the ear like a whole, in order that these worthless things [?], although they may be rough and crude, would at least yield matter for more profound speculation.

<PRIMA PARS>

G-ut prima clavis in manu, simplex atque gravissima nominatur. Quam clavem per lineam et non per spatium veteres signare decreverunt, ut potius ab habitu quam a privatione manum in cantu exordirentur.

Primam quidem ipsam locaverunt processum vitantes infinitum in descendendo; sic, ut et status fieret in ascensu, E-la clavem ultimam atque acutissimam statuerunt. Extra quas quidem claves fatuum esset ponere alias, cum omnem cantum congrue inter ipsas possibile sit disponi.

Simplicem eam dixi propter carentiam plurium sillabarum notas plures representantium. Sunt enim sex nomina notarum: ut, re, mi, fa, sol, la, et non plura; que, ut recitat Johannes ad Fulgentium in principio sue Musice, reperta fuerunt in primo versu ymni sancti Johannis *Ut queant laxis resonare*. Et sic se habent, quod nomen unius note mutari potest in nomen alterius, maxime dum distantia sex clavium prescise non sufficit cantus ordinationi, ut in antiphona *Alma redemptoris mater* et similibus. Sed cum huiusmodi mutatio ipsius note ut in quamvis aliam in dicta clave g-ut non sit necessaria, ideo simplicem obtinuit compositionem.

Gravissima eo dicitur, quia in gravitate locum primum meruit optinere. Et nota ut inter sex notas predictas in supradicto ymno inferior reperta exstitit, licet — forte propter illos, qui adeo subtiliter cantare nequeunt (non dico Alemanos, ne faciem propriam rubore perfundam verecundie) — cantus eiusdem ymni hodiernis temporibus exstat immutatus. Sed cum septem sint littere clavium representative: a, b, c, d, e, f, g (quarum una cognita consequenter cetere cognoscuntur, nec plus est necesse unam signare quam aliam, sed ad voluntatem cantum conscribentis), quero, quare potius veteres inceperunt manum a g littera quam ab a. Ad cuius dissolutionem taliter divino: ut nota ut, a qua manus inchoari debuit, per se sillabam

<FIRST PART.>

The first key in the hand, **g-ut**, is called simple and lowest. The old have decreed that this key must be marked on a line and not in a space, in order that in song, they could commence the hand from [a position of] *habitus* [possession, having] rather than *privatio* [privation, not having].

Indeed they have given the first [key] its place in order to prevent unending downward descent. Thus they [also] established the last and highest key of **E-la**, in order that there should be a limit to ascent. Outside of these keys it would be foolish to posit yet others, since every song can be properly disposed between [those two].

I have called [this key] simple because of the lack of more syllables representing more notes [than one]. For there are six names of notes: **ut, re, mi, fa, sol, la**, and no more. These, as Johannes says to Fulgentius at the beginning of his *Musica*, have been taken from the first verse of *Ut queant laxis resonare*, the hymn of St John. And they have this property, that the name of one may be changed into the name of another, especially when the range spanned by six keys does not suffice precisely for the disposition of a song, as in the antiphon *Alma redemptoris mater* and similar [chants]. Yet since this change of one note into any other is not necessary for the said key of **g-ut**, therefore it has retained simple composition.

It is called lowest because it has been worthy to receive the first place in lowness. And note that it is found at the lower end of the aforesaid six notes in the abovesaid hymn, even though the tune of the same hymn is different in today's times, perhaps because of those who cannot sing very subtly. (I am not speaking of Germans, lest I tinge [my] own face with the red color of shame). Yet since there are seven letters that are representative of the keys: **a, b, c, d, e, f, g** (and when you know one of these, you know the others; nor is it necessary to sign one more than another, but rather at the will of the one writing down the song), I ask wherefore the old began the hand with the letter **g** rather than **a**. I would guess that the resolution of this [question] is as follows: the note **ut** by itself, with which the hand has had to begin, makes a syllable just as the

faceret sicut in aliis quinque notarum nominibus. Si autem nota ut sequeretur litteram a, necessario in dyptongon aut coinciderent. Et ad consimilem preservandum, ne littera g cum nota ut unam sillabam generaret, fictum est, ut littera g sonet ut sillaba gam, sonum suum grecum observando. Cuiusmodi fictio aptius satisfiebat cum littera g quam quacumque alia clavis representativa, vel quia g littera prima fuit nominis ipsius auctoris, a qua littera sui nominis Guido Micrologum suum incepit, quem de musica edidit, quo litteram hanc dicit aliis adiuncatam per modernos.

Et quia g-ut est omnium clavium prima, cordam intendam in instrumento, que pulsata sonum reddat et primam notam, puta ut, cuius corde primum punctum ad modum monocordi Boetii signabo G simplici, que littera primam clavem manualem indicat; et gradatim per omnes claves cordam illam dividam, donec ad g secundum, puta g-sol-re-ut pervenero, ibidem usurus GG duplici, eo quod g-sol-re-ut ad g-ut dicitur clavis dupla. Contentabor insuper in divisione huiusmodi corde decem punctis dumtaxat tedium punctorum abhorrendo, cum cognitis illis decem de facili proportio cuiuslibet ceterarum clavium, ex quibus tota manus confecta est, ad aliam poterit indagari.

other five names of notes. But if the note **ut** were to follow the letter **a**, they would unavoidably fall together [and make] a diphthong. And in order to preserve the same thing [among keys] it was feigned, lest the letter **g** with the note **ut** would produce one syllable, that the letter **g** would sound as the syllable **gam**, observing its Greek sound. This fiction was more satisfactory with the letter **g** than with any other representative key. Or [perhaps] because **g** was the first letter of the name of the author himself, with which letter of his name Guido began his *Micrologus*, which he wrote about music, wherefore he speaks this neighboring letter to others, by people today.

And because **g-ut** is the first of all keys, I shall stretch a string on the instrument which, when struck, produces a sound [which is] the first note, namely **ut**. And the first point of this string I shall mark, after the manner of Boethius' monochord, with a single **G**, which letter indicates the first key of the hand. And I shall divide that string step by step through all keys, until I shall have arrived at the second **g**, that is **g-sol-re-ut**. And there I shall use double **GG**, because **g-sol-re-ut** is called the double key in relation to to **g-ut**. Moreover, I shall content myself with no more than ten points in the division of this string, being averse to tediousness of points, and with those ten known ones it shall be easily possible to find the proportion of any of the other keys from which the whole hand is made, relative to another.

Sit gratia exempli tota corda GK. Prescriptam cordam dividam in plures partes, sic tamen, quod partes eque intense se habeant ut prius: et visu percipiemus et auditu, quod ceteris paribus sicut se habebit tota corda ad eius partem seu partes, sic sonus totius corde ad sonum partis vel partium earundem.

Huius autem experientie primus repertor fuit Pitagoras in ponderibus quatuor malleorum, quos, ut certior esset, permisceri precepit, ne lacerti fabrorum quid adderent ad

For example, let the whole string be **GK**. I shall divide the afore-written string into several parts, but in such a way that the parts have the same tension as before. And we shall perceive by seeing and hearing that, just as the [length of the] whole string shall relate to its part or parts, so (other things being equal) [shall] the sound of the whole string [relate] to the sound of the part or of the same parts.

The first to find this out by experience was Pythagoras, in the weights of four hammers, which he directed to be exchanged in order that he could be more certain, lest the arms of the smiths would add something to the

sonorum proportiones eorundem. Ratio vero, quare brevior corda sonum reddit acutiorem, secundum Boetium et alios est, quia aerem celerius percutit. Et ideo pro experientia presupponamus: sicut una corda est altera in quavis proportione longior, sic sonus eius est sono alterius in eadem proportione gravior; et sicut una brevior, sic sonus eius acutior.

A-re secunda clavis est, re notam in se continens. Inter quam et g-ut tonus dicitur, et est clavis hec acutior quam illa in proportione sesquioctava, quod sic ostendo: Dividam GK cordam in novem partes equales, quarum partium sit GA prima; et sonabit AK tonum super GK, id est re super ut, quod ad experientiam relinquo auditus. Et quod proportio huiusmodi sonorum adinvicem comparatorum sit sesquioctava, probo per intellectum: quia tota corda GK continet semel AK et octavam partem ipsius AK, igitur per significationem vocabuli corda GK est in proportione sesquioctava longior parte sua, que est AK, ergo sonus eius in eadem proportione gravior et sonus ipsius AK in eadem proportione acutior per ea, que supposita sunt, et hoc est, quod volebam.

proportions of the same sounds. Yet the reason why the shorter string produces a higher sound, according to Boethius and others, is that it strikes the air more rapidly. And therefore we presuppose this for the experiment: just as one string is longer than another, by whatever proportion you like, so is its sound lower than the sound of another by that same proportion; and just as one is shorter, so is its sound higher.

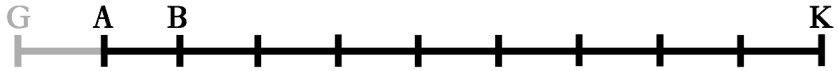
A-re is the second key, containing in itself the note **re**. Between it and **g-ut** [is the interval which] is called the tone. And this key is higher than the other by a sesquioctava proportion, 9:8. I demonstrate this as follows. I shall divide the string **GK** into nine equal parts, the first of which parts is **GA**. And **AK** shall sound a tone higher than **GK**, that is, **re** above **ut**. I leave this to the experience of the hearing. Now, that the proportion between these sounds, compared to each other, is sesquioctava, I demonstrate through understanding. Since the whole string **GK** contains both **AK** and the eighth part of this **AK**, therefore, by definition, the string **GK** is longer than its part **AK** by a proportion of 9:8. Hence its sound is lower by the same proportion, and the sound of this **AK** higher by that same proportion, [in accordance with] the things we have presupposed, which is what I wished [to demonstrate].

Et est proportio sesquioctava in genere superparticulari, sicut sunt omnes proportiones, quibus totum comparatur ad suam partem aliquotam, ut sunt sesquialtera, sesquitertia, sesquiquarta et sic in infinitum, ascendendo in denominatione, sed descendendo in significatione, nam quanto denominatio est maior in talibus, tanto proportio minor; est enim maior proportio sesquialtera quam sesquitertia. In numeris autem consistit sesquioctava proportio, in qua tonus quiescit, inter 9 et 8, eo quod numerus novenarius continet super numerum octonarium semel unitatem, que unitas est octava pars ipsorum octo; et vocatur pars aliquota, quia aliquotiens, puta octies, sumpta prescise reddit octo.

And the sesquioctava proportion is of the superparticular genus, just as are all the proportions in which the whole relates to its divisor, as are sesquialtera 3:2, sesquitertia 4:3, sesquiquarta 5:4, and so on, without end, becoming larger in denomination but smaller in meaning. For when the denomination is greater in such things, then the proportion is correspondingly smaller, as sesquialtera proportion is greater than sesquitertia. Yet in numbers, the sesquioctava proportion in which the tone resides is made up by 9 and 8, because the number 9 contains only one unity more than the number 8, which unity is the eighth part of those eight. And it is called a divisor, for when it is taken a number of times, like eight times, it produces exactly eight.

B-mi tertia clavis est, et est eius nota mi. Cuius si vis invenire sonum in respectu ad ut, divide predictam cordam AK in novem partes equales, et sit nona pars eius AB; pulsata vero BK dabit sonum mi super sonum corde AK. Nam equebene debet esse tonus inter re et mi, sicut fuit inter ut et re, ut voluerunt antiqui et repertum exstitit in dicto ymno Ut queant laxis et cetera; et omnes toni debent esse equales; necessarium igitur est, quod mi sonet duos tonos super ut, quod non solus auditus vel oculus, verum ratio demonstrabit.

B-mi is the third key, and its note is **mi**. And if you wish to find its sound in relation to **ut**, then divide the aforesaid string **AK** into nine equal parts, and let its ninth part be **AB**. But when struck, **BK** shall produce the sound of **mi** above the sound [made] by the string **AK**. For the tone between **re** and **mi** must be the same as that between **ut** and **re**, as the ancients wanted, and as it is found in the said hymn *Ut queant laxis*, and so on. And all tones must be equal. Therefore it is necessary that **mi** sounds two tones higher than **ut**, which not only the hearing and the eye, but also reason shall demonstrate.



Est namque proportio ipsius mi super ut, que dytonus dicitur, composita ex duabus sesquioctavis, quarum quelibet alteri est equalis, ex quo unum et idem nomen sortite sunt; ergo mi super ut faciet duos tonos, id est integrum dytonum, et latino nomine tertiam perfectam, sicut proportio GK ad BK componitur ex proportione, que est inter GK et AK, et ex proportione, que est inter AK et BK, quarum utraque sesquioctava vocata est.

Quod si vis scire nomen totius proportionis, que est inter mi et ut, oportebit querere tres numeros inequales, quorum maximus ad medium contineat proportionem sesquioctavam, et eandem medius ad minimum. Quod si ceperis 8 et 9, deficies in tertio, ex quo novenarius numerus non potest in octo partes dividi equales et per consequens non habet octavam partem. Multiplica igitur primo 8 per 8, id est 8 in se quadrato, et exibunt 64 (qui numerus ideo quadratus dicitur, quia in quadrato subtiliter positus facit relucere singula eius quadrati latera numero octonario); multiplica ultra numerum novenarium per 8, et exurgent 72; cui numero adde suam octavam partem, puta 9, et exibit numerus 81, qui comparatus ad numerum minimum, puta 64, dytonum sine dubio protexabit. Sunt enim ex huiusmodi multiplicatione et additione

For its proportion is **mi** above **ut**, which is called *ditonus* [the major third], and it is put together from two sesquioctavas, each being equal to the other, wherefore they have been given one and the same name. Hence **mi** above **ut** makes two tones, that is, a full third, and by its Latin name [called] *tertia perfecta*, just as the proportion **GK** to **BK** is put together from the proportion which is between **GK** and **AK** and the proportion which is between **AK** and **BK**, both of which are called sesquioctava.

Now, if you wish to know the name of the whole proportion between **mi** and **ut**, it will be necessary to find three unequal numbers of which the largest [relates] to the one in the middle in a sesquioctava proportion, and the one in the middle to the smallest in the same [proportion]. But if you take 8 and 9, then you shall lack the third [number], for which reason the number nine cannot be divided into eight equal parts, and consequently it does not have an eighth part. Therefore, multiply the first 8 by 8, that is, 8 squared, and this shall yield 64 (which number is called a square because in a square, when it is subtly made, it highlights each side of the square with the number 8). Thereafter multiply the number 9 by 8, and they shall result in 72. To this number [you must] add its eighth part, that is, 9, and this shall yield the number 81, which, when compared to the smallest number, namely 64, shall without doubt span the [interval of] the third. So by means of this multiplication and addition we have obtained three

reperti tres numeri inequales, videlicet 64 et 72 et 81, quorum sicut maximus ad medium constituit proportionem sesquioctavam, sic medius ad minimum; ergo proportio maximi ad minimum duas facit sesquioctavas et dytonum seu tertiam perfectam.

Vocaturque proportio huiusmodi dytoni superpartiens decem et septem sexagesimasquartas. Que proportio est in genere superpartienti, eo quod pars illa, quam maximus numerus continet supra minimum, non est pars aliquota ipsius minimi numeri, quoniam 81 continet ultra 64 17 unitates: que 17 unitates simul sumpte non faciunt partem aliquotam de 64, nam si quater capiantur, reddent 68 et sic plus, si vero ter capiantur, reddunt 51 et sic minus. Et faciunt superpartientes proportiones 5 ad 3, 7 ad 5, 9 ad 7, et sic ultra; vocantur autem speciali nomine superpartiens duas tertias ut 5 ad 3, alio nomine superbipartiens tertias, item proportio 7 ad 5 superpartiens duas quintas vel superbipartiens quintas, item proportio 9 ad 4 superpartiens quatuor quintas vel superquadripartiens quintas; et hec de hec.

Invenitur etiam tertia imperfecta ex tono et semitonio minore, de qua in fine huius opusculi plene dicam.

C-fa-ut quarta clavis manualis est, sed prima mixta, quia duas habet sillabas: fa et ut, que notas representant. Iuxta quod advertendum, quod hec clavis et quelibet alia mixta sub decima clave et eius dupla, que b-fa-♭-mi dicitur, de qua posterius tractabo, includit in se sillabas notarum, que seinvicem respiciunt in proportione sesquitertia; et si alias includit sillabas, hoc est propter b-fa-♭-mi, quia usque ad b-fa-♭-mi procedit tota manus per dyatessaron et exinde per tritonum et <ulterius> per dyatessaron, ut clare patet intuenti, sic quod dudum servatur dyatessaron propter eius dulcedinem et non totiens tritonus propter eius duriciem; propter dyatessaron igitur plus quam aliam quamvis proportionem predicte claves mixte talem retinent compositionem.

unequal numbers, namely, 64 and 72 and 81. And just as the largest of these relates to the middle one in a sesquioctava proportion, so the middle one [relates] to the smallest [in a sesquioctava proportion]. Therefore the proportion of the largest in relation to the smallest makes two sesquioctavas and [thus] the *ditonus*, or perfect third.

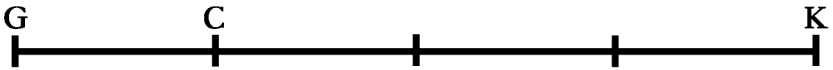
The proportion of such a third is called superpartient seventeen sixtyfourths. Which proportion is in the superpartient genus because the part which the greatest number has in excess to the smallest is not a divisor of that smallest number, because 81 contains 17 unities more than 64. Which 17 unities, taken together, do not make a divisor of 64, for if they are multiplied by four, they make up 68 and thus more [than 64], but if they are multiplied by three, they make 51 and thus less. And [examples of] superpartient proportions are 5 to 3, 7 to 5, 9 to 7, and so on. Yet they are called by a special name, like "superpartient two thirds" for 5 to 3, also, by another name, "superbipartient thirds," and again, "superpartient two fifths" or "superbipartient fifths" for the proportion 7 to 5, and also "superpartient four fifths" or "superquadripartient fifths" for the proportion 9 to [5], and so forth.

The imperfect third is found in a tone and minor semitone, of which I shall speak fully at the end of this little work.

C-fa-ut is the fourth key in the hand but the first composite one, because it has two syllables that represent notes: **fa** and **ut**. In which connection one should note that this key, and any other composites beneath the tenth key and its dupla called **b-fa-♭-mi** (which I shall discuss later on), includes within itself syllables of notes that relate to each other in sesquitertia proportion. And if it includes other syllables, this is because of **b-fa-♭-mi**, because up to **b-fa-♭-mi** the whole hand proceeds by the fourth and thereafter by the tritone and <beyond that> by the fourth, as will be clearly apparent to the observer, in such a way that the fourth is preserved before because of its sweetness, and the tritone not so many times because of its hardness; because of the fourth, therefore, the aforesaid composite keys retain their composition more than any other proportion.

Si itaque vis sonare fa super ut: divisa tota GK corda in quatuor partes equales, cuius quarta pars sit GC, sonabit autem CK fa super GK. Et habebunt se hii duo soni in proportione sesquitertia, quam dyatessaron constituit, qualis proportio sedet inter 4 et 3; nam tota corda GK continet semel CK et tertiam partem ipsius CK, quoniam pars illa, que est inter G et C, tanta est quanta una de tribus partibus ipsius CK. Et dicitur dyatessaron a 'dya', quod est 'de', et 'tessaron', quod est 'quatuor', quia ex quatuor constat clavibus et impossibile est illam ex pluribus paucioribusve disponi clavibus quam quatuor, ut posterius denarrabo.

So if you wish to sound **fa** above **ut**: having divided the whole string **GK** into four equal parts, of which the fourth part shall be **GC**, **CK** shall sound **fa** above **GK**. And those two sounds shall relate to each other by the proportion of sesquitertia, which constitutes the fourth and resides [in the relationship] between 4 and 3. For the whole string **GK** contains **CK** and the third part of this **CK**, because that part, which is between **G** and **C**, is equal to each of the three parts of this **CK**. And it is called *dyatessaron* after *dya*, which is "from," and *tessaron*, which is "four," because it contains four keys, and it is not possible to dispose it from more or fewer keys than four, as I shall explain below.



Ex predicta divisione se pandit semitonium minus inter fa et mi dominium suum tenens, de quo postea tractabo.

D-sol-re quinta clavis est, continens notam sol respectu re in a-re et re respectu sol in g-sol-re-ut. Et sic patet, quod ista sex notarum nomina sunt nomina relativa. Unde si non esset nisi unus sonus equalis et non diversificatus in cantu, non esset cura, quo nomine censeretur, ymo nullo nomine alicuius note potiretur, ut sonus campane vel alterius corporis non comparatus ad alium sonum.

Ad investigandum sonum sol super ut dividenda est tota corda GK in tres partes equales, quarum GD sit prima; resonabit utique GK notam ut et DK notam sol, videlicet dyapente, a 'dya', quod est 'de', et 'penta', 'quinque', quia de quinque clavibus constituta (alii tamen dicunt, quod ideo sic dicitur, quia ex quinque sectionibus habet fieri; nam corda, que graviorem eius sonum generat, dividitur in tres partes, corda, que acutiorem, in duas). Et est dyapente in proportione sesquialtera, ut inter 3 et 2, et vocatur latino nomine quinta.

The minor semitone extends between **fa** and **mi** in the aforesaid division, maintaining its dominion, which I shall discuss later.

D-sol-re is the fifth key, containing the note **sol** in relation to **re** in **a-re**, and **re** in relation to **sol** in **g-sol-re-ut**. And thus it is apparent that the six names of notes are relative names. So if there were only one identical sound, not diversified in song, like the sound of bells or another body that is not compared to another sound, then there would be no problem about the name it should be given, on the contrary, it would not have the name of any note.

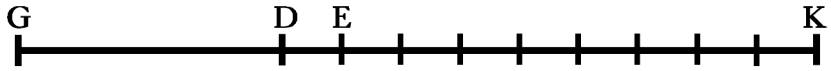
To inquire into the sound of **sol** above **ut**, the whole string **GK** must be divided into three equal parts, of which **GD** shall be the first. **GK** shall sound the note **ut**, and **DK** the note **sol**, that is, [together] the *dyapenthe*, [named] after *dya*, which is "from," and *penta*, "five," because it is made from five keys. (Others however say that it is so called because it is made from five sections; for the string which generates its lower sound is divided into three parts, and the string [which generates] the higher [is divided] into two.) And the fifth has the sesquialtera proportion, between 3 and 2, and it is called *quinta* by its Latin name.

E-la-mi sexta clavis est, duas habens notas, quarum quelibet cantatur secundum diversos respectus; continet enim la respectu mi in ♭-mi et mi respectu la in a-la-mi-re. Et est nota la acutissima omnium ipsarum sex, nec ultra ipsam quoquomodo possumus sine mutatione.

Et quia continet tonum super d-sol-re, divide DK cordam in novem partes equales, quarum sit DE prima. Habebit autem DK corda sesquioctavam proportionem ad EK, ergo tonum, sonabitque EK la super GK, puta e-la-mi super g-ut, et vocatur latino nomine sexta perfecta; perfectam dico, quia est dare sextam imperfectam constitutam ex dyapente et semitonio minore, ut postea videbis.

E-la-mi is the sixth key. It has two notes of which each is sung according to different relationships. For it contains **la** with respect to **mi** in **♭-mi**, and **mi** with respect to **la** in **a-la-mi-re**. And **la** is the highest note of all those six, nor can we go beyond it in any way without mutation.

And because it contains a tone above **d-sol-re**, divide the string **DK** into nine equal parts, of which **DE** shall be the first. The string **DK** shall however have the proportion of sesquioctava with respect to **EK**, and thus a tone, and **EK** shall sound **la** above **GK**, namely, **e-la-mi** above **g-ut**, and it is called a *sexta perfecta* by its Latin name. I call it perfect because it is superior to the imperfect sixth which is constituted from a fifth and a minor semitone, as you shall see later on.



Proportio vero ipsius la ad ut vocatur superpartiens undecim sextas decimas. Componitur namque ex proportione, que est inter sol et ut, que sesquialtera vocatur, ut 24 ad 16, et ex proportione, que est inter sol et la, que sesquioctava nuncupatur, qualis est inter 27 et 24; igitur proportio ipsius la ad ut est sicut 27 ad 16. Et quia 27 continet semel 16 et cum hoc 11 unitates de 16, patet nomen dicte proportionis.

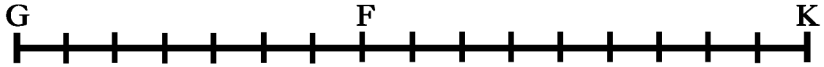
F-fa-ut septima dicitur; ad quam ultra e-la-mi ascendimus mutando la in mi. Ad quam clavem inveniendam super clavem precedentem dividenda est tota corda GK in sedecim partes equales, et contineat GF illarum partium 7 necesseque erit cordam FK continere residuas 9; et sonabit FK fa secundum supra GK, puta f-fa-ut supra g-ut, quorum sonorum proportio superpartiens septem nonas debeat vocari, qualem reperies inter 16 et 9, quoniam proportio, que est inter ut et secundum fa, componitur ex proportione, que est inter ut et primum fa, que sesquitertia dicitur, et ex alia, que est inter primum fa et secundum fa, que etiam sesquitertia est vocata, que, ex quo equales sunt, eodem nomine debent potiri, et per consequens

Now the proportion of this **la** to **ut** is called superpartient eleven sixteenths. For it is put together from the proportion which is between **sol** and **ut**, which is called sesquialtera, like 24 to 16, and from the proportion which is between **sol** and **la** which is called sesquioctava, which is between 27 and 24. Therefore the proportion of this **la** to **ut** is like 27 to 16. And since 27 contains one time 16 together with 11 unities of 16, the name of the said proportion is evident.

F-fa-ut is called the seventh [key], to which we ascend beyond **e-la-mi** by changing **la** into **mi**. To find the key above the preceding key, the whole string **GK** must be divided into 16 equal parts. **GF** should contain 7 of those parts, and it shall be necessary for the string **FK** to contain the remaining 9. And **FK** shall sound the second **fa** above **GK**, that is, **f-fa-ut** above **g-ut**. The proportion of this sound should be called superpartient seven ninths, which you find between 16 and 9. For the proportion between **ut** and the second **fa** is put together from the proportion between **ut** and the first **fa**, which is called sesquitertia, and from another which is between the first **fa** and the second **fa**, which is also called sesquitertia. And because they are equal, they must receive the same name, and in

componitur ex duabus sesquitertiis, velut proportio, que est inter 16 et 9, composita est ex proportione, que est 16 ad 12, et ex illa, que est 12 ad 9, quarum utraque sesquitertia nuncupatur.

consequence it is put together from two sesquitertias. Alternatively, the proportion between 16 and 9 is put together from the proportion between 16 and 12 and that between 12 and 9, both of which are called sesquitertia.



Et hec proportio minor est quam dupla in uno tono; nam si a dupla proportione resecetur sesquioctava, id est tonus, remanet proportio prius nominata superpartiens septem nonas; nam proportio 18 ad 9 componitur ex proportione 18 ad 16 et ex proportione ipsorum 16 ad 9; reseca ergo partem illam, que est 18 ad 16, in qua tonus consistit, remanebit pars maior, que superpartiens septem nonas est vocata, in qua se habet f-fa-ut ad g-ut.

Et dicitur hic fa respectu ut in c-fa-ut et ut respectu fa in b-fa-♭-mi.

G-sol-re-ut octava clavis est, et est mixta ex tribus sillabis notarum representanti<bu>s, et hoc propter respectum multiplicem. Continet namque sol respectu re in d-sol-re et re respectu sol in c-sol-fa-ut respiciendo bis ipsum c-sol-fa-ut; et sic duplicem habet respectum ad cantum superiorem, secundum quod ille cantus duriter cantari poterit vel molliter, id es per b-fa vel per ♭-mi, sed inferior solum uno modo. Et hec fuit ratio, quare alique claves duas habent notas et alique tres. Possunt autem fieri in hac clave quatuor mutationes, ut leviter patet intuenti.

Quod si vis audire hanc clavem super g-ut, adde supra f-fa-ut tonum, quem superius resecasti. Vel sic: divide totam GK cordam in duas partes equales, et sit medius punctus, puta punctus sectionis, GG, patebitque, quod medietas totius GK sonabit duplam supra totam cordam GK, et ita g-sol-re-ut super g-ut; propter quam rationem signam medium punctum sectionis duplici GG, quia duplam resonat proportionem ad primum g.

And this proportion is smaller than the dupla by one tone. For if a sesquioctava, that is, a tone, were cut from duple proportion, what remains is the aforenamed proportion superpartient seven ninths. For the proportion 18 to 9 is put together from the proportion of 18 to 16 and the proportion of these 16 to 9. Therefore, cut that part in which the tone consists, which is 18 to 16, and the greater part shall remain, which is called superpartient seven ninths, by which **f-fa-ut** relates to **g-ut**.

And it is called **fa** with respect to **ut** in **c-fa-ut** and **ut** with respect to **fa** in **b-fa-♭-mi**.

G-sol-re-ut is the eighth key, and it is a composite of three representative syllables of notes, and this because [there are] multiple relationships. For it contains **sol** with respect to **re** in **d-sol-re**, and **re** with respect to **sol** in **c-sol-fa-ut**, relating twice to this **c-sol-fa-ut**. And thus it has a twofold relationshp to the higher song, depending on whether that song is to be sung either hardly or softly, that is, through **b-fa** or through **♭-mi**, but only one [relationship] to lower song. And this was the reason why some keys have two notes and some three. For there are four mutations that can be made in this key, as is easily apparent to the observer.

So if you wish to hear this key above **g-ut**, add the tone above **f-fa-ut** which you have cut off above. Or thus: divide the whole string **GK** into two equal parts, and let the middle point, that is, the point of the section, be **GG**, and it shall be apparent that half of the whole **GK** shall sound the dupla above the whole string **GK**, and thus **g-sol-re-ut** above **g-ut**. For this reason I designate the middle point of the section with a double **GG**, because it sounds the dupla proportion above the first **g**.

Et vocatur proportio huiusmodi greco nomine dyapason, a 'dya', quod est 'de', et 'pan', quod est 'totum', et 'sonus', 'soni', quia includit in se totum sonum, id est omnes gradus sonorum, per quos cantus habet ordinari, quod evidenter patet ex hoc: nam si volumus ascendere ultra G-sol-re-ut ad altiores claves, operemur eodem modo circa secundum medietatem totius GK corde, quo facimus circa ipsam totam, videlicet dividendo secundam medietatem in novem partes equales, et facient octo illarum partium tonum supra totam medietatem et per consequens tonum cum dyapason ad totam cordam GK, et hoc modo procede gradatim usque ad quadruplam dividendo totam cordam in quatuor partes, quarum ultima quarta sit HK, quam invenies sonare quadruplam super totam GK, puta secundum g-sol-re-ut super g-ut, id est g tertium super g primum. Et quia cantus supra dyapason non est nisi iteratio cantus infra dyapason cum additione nominis duple vel alterius ad duplam multiplices, ideo in clave g-sol-re-ut littere clavium precedentium reiterantur.

Si etiam vis audire triplam proportionem, id es d-la-sol-re super g-ut, divide totam GK in tres partes equales; sonabit autem quelibet eius tertia super totam GK d-la-sol-re super g-ut. Vel sic: divide secundam medietatem totius GK corde in tres partes, et sonabunt due eius partes simul sumpte super totam medietatem d-la-sol-re super g-sol-re-ut, id est sesquialteram proportionem; facit autem g-sol-re-ut duplam super g-ut, igitur d-la-sol-re super g-ut triplam, que ex dupla et sesquialtera censetur constituta. Redeunt autem hee due practice operationes in unam, quoniam due tertie alicuius medietatis valent unam tertiam totius.

— Ex quo concluditur, quod duodecima clavis non debet vocari sexta duplata vel quinta duplata, prout multi credunt, sed vocabitur quinta supra duplam, et magis proprie vocatur tripla. Nec debet triplum cuiuscumque moteti vocari hoc nomine, nisi in ipso sit aliqua nota triplam habens proportionem ad notam

And this proportion is called *dyapason* in Greek, from *dya* which is "from," and *pan* which is "the whole," and *sonus, soni*, because it encompasses the whole sound, that is, all steps of sounds by which song is to be arranged, which is clearly apparent from [the following]. For if we wish to ascend beyond **G-sol-re-ut** to higher keys, we would operate in the same way in the second half of the whole string **GK** as we [did] in the whole, namely, divide the second half into nine equal parts, and eight of those parts shall make a tone above the whole half, and consequently the tone plus octave to the whole string **GK**. And proceed in this way, step by step, up to the quadruple, dividing the whole string into four parts, of which the last fourth would be **HK**, which (you will find) sounds the quadruple above the whole **GK**, that is, the second **g-sol-re-ut** above **g-ut** or the third **g** above the first **g**. And since song above the octave is nothing but a repetition of song beneath the octave, except for the addition of the name of dupla or other multiples of dupla, therefore the letters of the preceding key are repeated in the key of **g-sol-re-ut**.

And if you wish to hear the triple proportion, that is, **d-la-sol-re** above **g-ut**, divide the whole **GK** into three equal parts; now any third of them shall sound, above the whole **GK**, **d-la-sol-re** above **g-ut**. Or like this: divide the second half of the whole string **GK** into three parts, and its two parts taken together above the whole half shall sound **d-la-sol-re** above **g-sol-re-ut**, that is, the sesquialtera proportion. But **g-sol-re-ut** makes the dupla above **g-ut**, therefore **d-la-sol-re** above **g-ut** the tripla, which is judged to be constituted from the dupla and sesquialtera. But those operations come down to one in practice, because the two thirds of any half are worth one third of the whole.

— From which it may be concluded that the twelfth key must not be called *sexta duplata* or *quinta duplata*, as many believe, but rather it should be called the fifth above the dupla. And it is more properly called tripla. Nor must the triplum of any motet be called by this name, unless it contained some note that has the tripla proportion

aliquam, puta moteti vel tenoris. Sic suo modo dicendum est de decima clave ac aliis. —

Quod si delectat scire, qualis proportio sit inter primam et ultimam claves ipsius manus, puta inter g-ut et e-la, divide ultimam quartam totius GK corde in 27 partes equales, quarum contineat corda HI 16; resonabit HI perfectam sextam supra HK, id est proportionem superpartientem undecim sextasdecimas, qualis est inter e-la et secundum g-sol-re-ut; sed HK sonat quadruplam super totam GK, videlicet secundum $\frac{\text{gg}}{\text{gg}}$ super g-ut; igitur HI se habet ad totam GK sicut e-la ad g-ut in proportione sextupla superpartiente tres quartas, qualiter in numeris reperitur inter 27 et 4; que minor est quam septupla, que inter 28 et 4 existet et a musica refutatur.

with respect to another note in the motetus or tenor. The same is to be said in its way about the tenth key and others. —

And if [the reader] wishes to know the proportion between the first and last keys of the hand, that is, between **g-ut** and **e-la**, then [you must] divide the last fourth of the whole string **GK** into 27 equal parts, of which the string **HI** contains 16. **HI** shall sound the perfect sixth above **HK**, that is, the proportion of superpartient eleven sixteenths, which is between **e-la** and and the second **g-sol-re-ut**, namely the second $\frac{\text{gg}}{\text{gg}}$ above **g-ut**. Therefore **HI** relates to the whole **GK** as **e-la** to **g-ut** by the proportion of sextupla superpartient three fourths, which in numbers is found between 27 and 4. This is smaller than the septupla, which exists between 28 and 4, and is rejected by [the art of] music.

<SECUNDA PARS>

Tonus, qui grece epogdus dicitur, secundum Boetium et omnes non potest dividi in partes equales.

Videtur tamen, quod ymno, quia omnis proportio secundum Euclydium se habet ut linea; nam sicut una linea longior est alia, sic una proportio est alia maior: modo quelibet linea potest dividi per medium.

Item si due corde se habeant in proportione equalitatis et intendatur altera continue usque ad proportionem sesquioctavam, non est dubium, quin fiet transitus per medium.

In materia ista, quamvis in consimili argumento circa quadraturam circuli probandam Brysso seu Ypocias seu quicumque alius fuerit per Aristotelem primo Elencorum per quatuor folia ante finem reprobatu<s>, credo igitur quod supradicta argumenta concludunt, sed dico dictum Boetii intelligendum fore, videlicet quod tonus non dividitur in partes equales stante armonia; et hoc idem addit Boetius, quando de

<SECOND PART.>

According to Boethius and all [others], the tone, which in Greek is called *epogdous*, cannot be divided into equal parts.

However, it seems that the contrary is true, for every proportion, according to Euclid, is like a line. And just as one line is longer than another, so is one proportion greater than another. And every line can be divided through the middle.

Also, if two strings are related to each other in a proportion of equality, and if one of them were to be gradually stretched to the sesquioctave proportion, there is no doubt that there would be a transition through the middle.

Therefore, although Aristotle, in a similar argument about proving the squaring of a circle, condemned Bryson or Hippocrates or somebody else in the first book of his *Sophistical Refutations* (four leaves before the end), I believe, in the matter at hand, that the above arguments are valid. Yet I say that the statement of Boethius, namely, that the tone is not divided into equal parts with harmony remaining, shall have to be [properly] understood. When Boethius spoke of the division of the tone, he also

divisione toni locutus est, nam dicit expresse, quod dyesis et scisma veram medietatem toni constituunt (dyesis, id est semitonium minus, quod apud antiquos lyma vocabatur; scisma est medietas commatis).

Sed ad experientiam probatur, si aliqua esset corda media, que se haberet ad cordam AK sicut GK ad illam, quod sonus illius non faceret gradum aliquem comparatus ad sonum aliarum cordarum, quia usque ad hec tempora non placuit talis cantus saltem a nobis prolatus, qui dyatonico generi insistimus (de cantu seu armonia corporum celestium et angelorum seu volucrum nescire, divinare). Sic nec duo soni, qui se in medietate habent duple proportionis, accepti sunt nec simul, nec separatim, qualem facerent due corde, quarum una se haberet ut dyameter, alia ut costa sui quadrati, et sic de aliis, que omnia experientie relinquo auditus.

added this point, and said expressly, that the diesis and schisma together constitute the true middle of the tone (The diesis, that is the minor semitone, which was called limma by the ancients. The scisma is half of a comma).

Yet if there is some middle string which relates to the string AK just as GK [does] to it [that is, $\sqrt{9}:\sqrt{8}$], its sound would not represent a step comparable to the sound of the other strings, as experience demonstrates. Because up to the present time song of this kind, at least as performed by us who insist on the diatonic genus, has not been pleasing. (I cannot tell anything about the song or harmony of heavenly bodies and of angels or of birds.) But neither are two sounds that relate by half of the duple proportion [$\sqrt{2}:1$] well liked, whether simultaneously or separately. This kind [of proportion] would be made by two strings of which one was like the diameter of a square, and the other like a side, and thus also the others, all of which I leave to the experience of the hearing.

Rationem tamen istorum aliqui nituntur signare, quod hoc eo fit, quia musica applaudit arithmetice, que inter numeros versatur, in qua omnes proportiones rationales, id est aliquo modo commensurabiles, existunt, et non geometrice, in qua irrationales inveniuntur proportiones, ut est dyametri ad costam. Modo proportio sesquioctava non habet mediam proportionem in numeris, nam inter 9 et 8, qui primi numeri sunt proportionis sesquioctave, non cadit medius numerus equali proportione distans ab extremis, igitur nec inter ipsos, qualitercumque multiplicentur: unde inter bis octo et bis novem cadit numerus 17, qui non equali proportione respicit utrumque, nam maior est proportio 17 ad 16 quam 18 ad 17. Eodem modo inter 2 et 1 non cadit numerus medius; igitur in numeris non inveniuntur aliqui duo numeri, qui seinvicem aspiciant in medietate duple proportionis.

Some, however, seek to determine the reason for this, which is that music pleases mathematics which deals with the numbers in which all rational proportions exist (that is, [numbers which are] in some way commensurable), and not geometry in which irrational proportions are found, such as between the diameter and the side [of a square]. Yet the sesquioctava proportion does not have a middle proportion in numbers. For between 9 and 8, which are the first numbers of the sesquioctava proportion, there is no middle number that is equally distant, by the same proportion, from both extremes. And no matter how many times [those extremes] may be multiplied, still [there is] no [number] between them. Thus, the number 17 lies between twice eight [16] and twice nine [18], yet it does not relate to both with the same proportion. For the proportion 17:16 is greater than 18:17. In the same way there is no middle number between 2 and 1 [in dupla proportion]. Therefore one cannot find any two numbers, among the [integer] numbers, that relate to each other by half of the duple proportion.

Item quod tonus et quelibet alia superparticularis proportio caret medio proportionali, patet ex hoc, quia talis medius numerus esset radix, id est latus tetragonale, numeri producti ex numerositate extremorum in se, sicut numerus senarius est medius inter 4 et 9: multiplicatur igitur 9 per 4, exeunt 36, numerus quadratus, cuius senarius est latus, ut patet satis in figuris Arithmetice. Modo extrema proportionis superparticularis in se ducta non procreant numerum quadratum, quod satis alibi est demonstratum.

Sed quamvis hec certissima sint, non tamen credo, quod hec est ratio sufficiens in proposito nisi quodammodo a posteriore. Nam presupponitur, quod nulla proportio duorum sonorum possit facere gradum congruum in ascensu vel descensu, nisi illa proportio in numeris signari possit (de quo satis dubito, utrum saltem possibile sit huiusmodi proportionem invenire vel gradum). Nam secundum diversitatem temporis et regionum multa nova et inaudita poterunt suboriri, sicut forte pronuntiatio commatis et trium semitoniorum minorum ac multorum similium, que, licet hactenus non audita sunt, forte tractu temporis per nova instrumenta et vocum habilitates posterius audientur, sicut nec ante Pitagoram fuit tanta subtilitas in cantu, quanta hodiernis temporibus est in usu, nec talem nos, qualem Anglici, G<alli>ci vel Lumbardi in cantu facimus fracturam.

Item licet ponat Boetius minoris semitonii medietatem arismetricam, que neglecta proportione equalitatem differentiarum considerat, eius tamen non ponit medietatem geometricam, de qua querimus, que inter numeros non reperitur; et nichilominus eius semitonii proportionalis medietas, id est geometrica, saltem in cantu enarmonico apud antiquos gradum fecit (cuius quidem cantus tetracordum per dyesim et dyesim processit et dytonum; et existit dyesis apud ipsos vera lymmatis medietas); bene tamen assentio, quod

That the tone, and any other superparticular proportion, lacks a proportional middle is apparent also from this, namely, that such a middle number would be the root (that is, the side of a square) of the number that is produced by the multiplication of the extremes with each other, like, for example, the number 6 is the middle between 4 and 9. For if 9 is multiplied by 4, they produce 36, a square number, of which the number six is a side, as is sufficiently apparent from the figures in arithmetic. But when the extreme [numbers] of the superparticular proportion are [multiplied] with each other, they do not produce a square number, as has been sufficiently demonstrated elsewhere.

Yet although these things are indeed most certain, I do not believe that this is sufficient reason in the case at hand, except in some way from a later perspective. For it is presupposed that there is no proportion between two sounds that can make a harmonious step in ascent or descent, unless that proportion is expressible in numbers. (For this reason I doubt whether it might be even possible to find such a proportion or step.) On the other hand, it is possible that many new and unheard things will come to be, according to the diversity of times and regions, such as, perhaps, the performance of the comma, and of three minor semitones, and of many similar things. Even though these have not been heard up to now, they may perhaps be heard in the future, after some lapse of time, by means of new instruments and abilities of voices, just as there was not so much subtlety in song before Pythagoras as is in use nowadays. Nor do we apply fracturing [of notes] in song quite like the English, the French, or the Lombards do.

And although Boethius posits a mathematical half for the minor semitone (which [half] concerns the equality of different things, disregarding proportion), he does not posit the geometrical half about which we are inquiring here, and which is not found in numbers. Nevertheless the proportional half of the semitone, that is, the geometrical one, has represented a step at least in the enharmonic song of the ancients. (In their singing, indeed, the tetrachord proceeded by diesis and diesis and major third, and among them the diesis is truly half of the limma.) But I do agree that the step, by which we above all

quidem gradus, quo nos maxime dyatonici hactenus usi fuimus in musica, possit proportione arismetrica mensurari.

Lima apud antiquos est minus semitonium apud nos, intervallum scilicet de fa ad mi. Quod ideo minus dicitur, quia medietatem toni, prout patet ex predictis, non attingit, quoniam inter primos numeros sesquioctave proportionis non est numerus medius, qui equaliter participet extremos.

Item ex quo dyapason, ut in suprascripta corda GK leviter descerni potest, constituitur ex quinque tonis cum duobus semitoniis minoribus, que duo semitonia minora si complerent tonum prescise, sequeretur, quod tres toni facerent medietatem duple proportionis, et sic tritonus mediaret dyapason, quod est impossibile.

Captis enim tribus numeris, illis videlicet, quos supra in clave ♭-mi cepimus, puta 64 et 72 et 81, quorum quilibet multiplicetur per 8, addaturque tertio producto ex huiusmodi multiplicatione sua octava et fiat quartus: faciet infallibiliter quartus ad primum proportionem tritoni.

Verbi gratia: multiplicatur 64 per 8, provenient 512, qui numerus sit primus; multiplicatur vero 72 per 8, resultant 576, qui sit secundus; sic et octies 81 reddunt 648, qui sit tertius; cui tertio adde suam octavam partem, et pariet 729, qui sit quartus: inter hos quatuor proportionales numeros sunt tres sesquioctave, ergo et tres toni, et ergo maximus, puta 729, comparatus ad minimum, puta ad 512, facit revera tritonum (quorum maior continet ultra minorem 217 unitates; vocatur ideo proportio tritoni superpartiens ducentas et decem et septem quingentesimas duodecimas). Si ergo tritonus esset medietas ipsius dyapason, id est proportionis duple, sequeretur, quod numerus minimus duplatus, puta 1024 se haberent ad 729 sicut 729 ad 512 et quod equalis esset proportio utrobique, quod non est verum, quia est minor maximi ad medium quam medii ad minimum in quantitate commatis, ut infra proximo capitulo ostendetur.

have used so far the diatonic ones in music, may indeed be measured by an arithmetical proportion.

The limma of the ancients is the same as our minor semitone, that is, the interval between fa and mi. It is called minor because it does not fully attain half of the tone, as is apparent from what has been said before. For there is no middle number in between the first numbers of the sesquioctava proportion, that could divide the extremes equally.

It is also for this reason that the octave is constituted from five tones with two minor semitones, as can be easily observed in the abovewritten string GK. But if these two minor semitones were to complete a tone exactly, it would follow that three tones constituted half of the duple proportion, and that the tritone would be the middle of the octave, which is impossible.

For when we take three numbers, namely, the ones taken above in the key ♭-mi, that is, 64 and 72 and 81, and if we multiply each of these by 8, and we add to the third product of this multiplication its own eighth part, then we obtain a fourth [number]. And the fourth [number] shall make, without fail, the proportion of the tritone with the first [number].

For example: 64 multiplied by 8 makes 512, which is the first number. And 72 multiplied by 8 makes 576, which is the second. Likewise eight times 81 makes 648, which is the third. Now add, to this third [number], its own eighth part, and it shall make 729, which is the fourth [number]. Among these four proportional numbers there are three sesquioctava [proportions], and therefore three tones. Therefore the largest, that is 729, does indeed make a tritone in relation to the smallest, that is, to 512. (The numerator of this tritone contains 217 unities more than the denominator; therefore the proportion of the tritone is called superpartient 217 512ths). If, therefore, the tritone were half of the octave, that is, of dupla proportion, it would follow that the least number doubled, that is, 1024, would relate to 729 as 729 relates to 512, and that there would be the same proportion in both cases. This is not true, however, for [the proportion] between the largest and middle [numbers] is less than that between the middle and smallest [numbers], by the quantity of a comma, as will be shown below in the next chapter.

Relinque igitur, quod distantia inter fa et mi minor est quam medietas toni.

Ex quo possunt extrahi conclusiones tales: quod dyapason non continet sex integros tonos; item quod duo semitonia minora simul sumpta non mediant tonum, quia tunc, cum quatuor talia integrarent tonum, sequeretur, quod sesquioctava proportio haberet numerum medium; ex quod proportio semitonii minoris tamen in numeris reperitur, ut subsequenter ostendam; item quod duo semitonia minora plus faciunt quam medietatem toni, nam medietatem non faciunt per ea, que dicta sunt, si igitur minus facerent, tunc quatuor subintegrarent tonum, quod esse non potest. Consimiles conclusiones potest, quis subtilis, per se ex hiis, que dicta sunt, facilius extorquere.

Numeri autem primi, inter quos minus semitonium inventum est, sunt 243 et 256, qui adinvicem se respiciunt in proportione superpartienti tredecim ducentesimas quadragesimas tertias. Possumus dictos numeros semitonii taliter venari: Certum est enim, ut supra patuit in clave c-fa-ut, dyatesseron constare ex dytono et semitonio minore. Numeri primi ipsius dytoni, ut patuit in ♭-mi, sunt 81 et 64. Modo non est dare numerum, qui possit se habere ad 64 in proportione sesquitertia, in qua dyatesseron quiescit, ex quo 64 non possunt dividi in partes tres equales: multiplicabo igitur ipsum per numerum ternarium, et exibit numerus 192, qui numerus, eo quod per ternarium multiplicatus est, optime divisibilis est in tres partes. Multiplicatis eodem modo 81 patebit numerus 243, manebitque proportio dytoni inter 243 ex una parte et 192 ex alia. Addatur minori numero sua tertia pars, et veniet numerus 256, qui comparatus ad numerum 243 facit proportionem semitonii minoris, quam querebamus.

Resecato vero dytono a dyatesseron remanet lima, minus semitonium, quod greco nomine dyesis secundum Phylolaum dicitur, licet Boetius vocet dyesim minoris semitonii

Therefore, leave [to be concluded] that the distance between fa and mi is less than half a tone.

From [all] this we can draw the [following] conclusions. [First,] that the octave does not contain six whole tones. Also that two minor semitones taken together do not constitute half a tone. For if four such [minor semitones] could make a whole tone, it would follow that the sesquioctava proportion did, in fact, have a middle number. From this [it is apparent], however, that the proportion of the minor semitone is [expressible] in numbers, as I shall show later on. Also that two minor semitones make up more than half a tone, since they do not quite make up half, because of what we have said. And if they would make less, then four [minor semitones] would make up less than the whole tone, which cannot be. From the things that have been said, a person of subtle [mind] can draw similar conclusions more easily for himself.

Yet the first numbers of [the proportion of] the minor semitone are 243 and 256, relating to each other in the proportion of superpartient thirteen 243rds. We can find the said numbers of the semitones in this way. It is certain, as was apparent above in the key c-fa-ut, that the fourth consists of a major third and a minor semitone. The first numbers of this third, as was apparent in ♭-mi, are 81 and 64. But there is no number that can relate to 64 in a sesquitertia proportion, [that is, the proportion] in which the fourth resides, because 64 cannot be divided into three equal parts. I shall therefore multiply it by the number three, and this will yield 192. Since this number was multiplied by the number three, it is perfectly divisible into three parts. Having multiplied 81 in the same way, we shall arrive at 243. And between 243 on the one hand, and 192 on the other there shall be the proportion of a major third. Now we add, to the smaller number [192], its own third part [64], and we shall have the number 256, which makes the proportion of the minor semitone with the number 243, which is what we were looking for.

When we cut the major third from the fourth, what remains is the limma, the minor semitone, which according to Philolaus is called *diesis* in Greek, even though Boethius says that the diesis is half of the minor

medietatem, que medietas alio vocabulo dyascisma nuncupatur.

Comma secundum Boetium et alios est excessus ille, in quo sex toni excedunt dyapason, et per consequens est excessus ille, in quo tonus excedit duo semitonia minora et maius semitonium minus.

Proportio autem commatis consistit inter hos numeros: 524288 ex una parte et 531441 et altera, et iste excedit illum in 7153 unitatibus, unde posset cito colligi nomen proportionis, si grammatice non obstarent.

Quos numeros visa practica circa inventionem numerorum semitonii minoris poteris leviter per multiplicationes investigare: dispositis tribus numeris taliter, quod maximus ad minimum contineat spatium sex tonorum prescise, erit necessario inter eos numerus, qui ad minimum comparatus faciet dyapason; cuius, inquam, dyapason proportione subtracta a proportione sex tonorum remanebit nudum et solum comma.

Quia dum verbi gratia tritonus, ut dixi supra capitulo proximo, clauditur inter 729 ex una parte et 512 ex alia, que proportio tritoni si dupletur, resultabit proportio sex tonorum. Ad quam ergo duplandam queram tertium numerum, ad quem maior numerus ipsius tritoni se habebit sicut minimus ad ipsum. Cum autem hoc non potest fieri in numeris primis ipsius tritoni, multiplicabo ipsos sic, quod ille maior, qui medius erit, divisibilis sit in quingentas et duodecim partes equales, nam opportet, quod inter ipsum et tertium numerum remaneat denominatio proportionis ipsius tritoni.

Ut a simili: si vis habere numerum, qui eandem habeat proportionem ad 5, quam habent 5 ad 3, non illum numerum reperies, nisi multiplices 5 per numerum ternarium, et exibunt 15, et eodem modo 3, et producetur numerus novenarius; quo facto adde numero 15 duas tertias suiipsius, puta 10, et exibunt 25; erit autem proportio 25 ad 15 sicut 15 ad 9. Et hanc practicam docet denominatio proportionis, quia proportio inter

semitone, which half is called diaschisma by another name.

According to Boethius and others, the comma is the amount by which six tones exceed the octave, and that amount, consequently, is that by which the tone exceeds two minor semitones, and the major semitone the minor.

Yet the proportion of the comma is made up of these numbers: 524,288 on the one hand, and 531,441 on the other, and the latter exceeds the former by 7153 unities. One mightly quickly deduce the name of this proportion if only [those numbers] did not defeat grammar.

Having seen the practical [steps] by which the numbers of the minor semitone can be found, you shall now easily be able to investigate those numbers by means of multiplications. Having disposed three numbers in such a way that the largest contains the exact span of six tones relative to the smallest, we shall now need a number between them which shall make an octave relative to the smallest. And I say that if the proportion of this octave is subtracted from the proportion of six tones, there shall remain a bare and sole comma.

For since the tritone, as said above in the previous chapter, is enclosed between 729 on the one hand and 512 on the other, then if that proportion of the tritone is doubled, what will result is the [cumulative] proportion of six tones. In order to double that [proportion], therefore, I seek a third number, to which the larger number of the tritone [729] shall relate as the smaller [512] does to it. But since we cannot do this with the first numbers of this tritone, I shall multiply those [numbers] in such a way that the larger number, which shall become the middle, is divisible into 512 equal parts. For it is necessary that between it and the third number, the denomination of the proportion of this tritone remains the same.

To cite a similar case: if you wish to have the number that has the same proportion relative to 5 as 5 has to 3, you shall not find that number unless you multiply 5 by the number 3, which makes 15. And in the same way [you must multiply] 3, and it shall produce the number 9. Having done this, add two thirds to the number 15, that is, 10, and it makes 25. And the proportion of 25 to 15 shall be like that between 15 and 9. And the denomination of the proportion instructs this practice, because the

5 et 3 vocatur superpartiens duas tertias; et sicut nomen trahit a duabus tertiis, sic in proposito proportio tritoni nomen trahit a ducentis et decem et septem quingentesimis duodecimis.

Ergo per illum numerum 512 multiplicandus est uterque numerus ipsius tritoni, ut quilibet divisibilis sit in totidem, puta 512, partes equales multiplicato ergo dicto numero, scilicet 512, in se, veniet numerus 262144. Multiplicatis vero 729 per 512 procedet numerus 373248. Inter quos est tritonus. Cuius, inquam, numero maiori adde ducenties decies et septies suam quingentesimam duodecimam, puta 158193 unitates, et orietur numerus tertius, videlicet 531441. Qui tres numeri adinvicem sunt continue proportionales, nam inter maximum et medium est tritonus sicut inter medium et minimum; et ergo maximus ad minimum facit proportionem sex tonorum.

Quo minimo numero duplicato lucebit numerus 524288, qui comparatus ad minimum faciet dyapason. Cuius dyapason proportione reiecta manet inter 531441 ex una parte et 524288 ex altera ipsius nudi et solius commatis proportio, quam querebamus. Et est comma minima distantia duorum sonorum in cantu possibilis, ut innuit Boetius, cum sic ait: *Comma est ultimum auditui subiacens ultimaque proportio*. Et quamvis dicat Boetius, quod comma dividitur in duo scismata, ubi de divisione toni loquitur, non tamen vult, quod scisma faciet aliquem gradum in musica, nam auris adeo modicam distantiam sonorum forte non potest percipere; licet etiam perciperet, non tamen sequitur, quod scisma gradum faceret, sicut nec medietas duple proportionis, que bene perceptibilis est, nec medietas ipsius dyesios, id est semitonii minoris, quam medietatem dyascisma Boetius appellat.

Sed queres forte, quare potius in musica fiat mentio de commate quam de dyascismate? Respondetur per ea, que dixi supra, quod hoc ideo est, quia diascisma, prout est vera medietas minoratis, non in numeris invenitur, sicut bene comma, et musica subordinatur arismetrice. Sed hec responsio

proportion between 5 and 3 is called superpartient two thirds. And just as it derives its name from two thirds, so does the proportion of the tritone, in the matter at hand, draw [its] name from 217 512ths.

Therefore both numbers of this tritone are to be multiplied by the number 512, and both shall be divisible by as many equal parts, that is, 512. And having multiplied the said number, namely 512, with itself, the resulting number shall be 262,144. And having multiplied 729 by 512, we arrive at the number 373,248. And there is a tritone between these [two numbers]. Add 217 times [729], that is, 158,193 unities, to the larger number, and what results is the third number, namely 531,441. These three numbers are uniformly proportional to each other, for between the greatest and middle [number] is a tritone, just as between the middle and the smallest [number is a tritone]. Therefore the largest makes the [cumulative] proportion of six tones relative to the smallest.

If the smallest number is multiplied [by itself], the number 524,288 shall appear, which is the octave relative to the smallest [number]. Now when this proportion of the octave is removed [from the proportion of the double tritone], there remains the proportion of the bare and sole comma, which is between 531,441 on the one hand and 524,288 on the other. And the comma is the smallest possible distance between two sounds in song, as Boethius intimates when he speaks thus: *The comma is the last thing still subject to the hearing and the last proportion*. And although Boethius, in the place where he speaks of the division of the tone, says that the comma is divided into two schismata, he does not mean that the schisma therefore makes a step in music. For the ear cannot perceive so small a difference between sounds. And even if would perceive it, it does not follow that the schisma makes a step, just as half a duple proportion, which [as such] is perfectly perceptible, [does not make a step either], nor half the diesis, that is, the minor semitone, which Boethius calls the diaschisma.

Yet perhaps you will ask, why is the comma more frequently mentioned in music than of the diaschisma? Let the answer be what I said above, [namely,] that the [proportion of the] diaschisma, inasmuch as it is the true half of the minor semitone, is not found among numbers. But the comma is, and music is subordinate to mathematics. Still, this response

non sufficit, nam in numeris infinite sunt proportiones, quas musica non advertit. Ideo aliter potest dici, quod hoc est per accidens, quod musica loquatur de comate, nam ex quo dyapason minus continet quam sex tonos, et non plus quam quinque cum duobus semitoniis minoribus. Ergo per consequens et accidentaliter, non autem primo et per se discutit musica differentiam sex tonorum supra dyapason, non autem quod comma faciet gradum musicalem, qui hactenus inter nos fuerit usitatus, nam cantus comatis neque dyatonicus neque cromaticus nec enarmonicus ex sufficienti divisione tetracordorum Boetii fore comprobatur, quamvis bene inter claves manuales re<p>peri coma possibiliter disponendum, quod dicam infra in clave b-fa-♭-mi.

Apothome grece latine maius semitonium est; quod et ideo maius dicitur, quia plus continet quam veram medietatem toni: quoniam constat ex semitonio minore et comate, igitur plus continet quam semitonium minus; sed semitonium maius et semitonium minus constituunt prescise tonum; si igitur semitonium maius esset vera toni medietas, — cum unumquodque totum constuitur ex duabus suis medietatibus — sequeretur, quod semitonium minus esset alia toni medietas, quod falsum est, ut iam est ostensum.

Ante quos autem numeros semitonium maius est applexum, visis hiis, que dicta sunt de semitonio minore et commate, per te satis indagabis. Qualiter vero maius semitonium inter claves manuales disponitur, in b-fa-♭-mi pertractabo.

Octo commata et amplius, minus tamen quam novem continet ipse tonus, quod per Boetium leviter demonstratur. Et sit A numerus 262144, et sit B numerus distans ab A spatio quinque tonorum, puta 472392; sit autem C numerus duplus ad A, id est dyapason, scilicet 524288, D vero numerus, distans ab A spatio sex tonorum, 531441. Distat ergo D a numero C solo commate, quanto spatio sex toni distant a dyapason: que quidem differentia, id est excessus ipsius D ad C, sit E, puta 7153.

does not quite suffice, for among numbers there are infinitely many proportions that are not regarded in music. Therefore it may be stated differently, namely, in an accidental sense, that music speaks of the comma because this is the [proportion] by which the octave is less than six tones. And it does not exceed five [tones] plus two minor semitones. In consequence, therefore, music does not primarily and for its own sake, but rather in an accidental sense, deal with the difference between six tones and the octave. So it is not as if the comma makes a musical step that we would have used to this day. For the song of the comma cannot be shown to be either diatonic, nor chromatic, nor enharmonic, in any adequate division of Boethius's tetrachord, even though it is possible to arrange for the comma to be found among the keys of the hand, as I shall say below under the key of b-fa-♭-mi.

Apothome, in Greek, is [called] *maius semitonium* [major semitone] in Latin. It is called major because it contains more than the exact half of a tone. For it consists of the minor semitone plus comma, and therefore it contains more than the minor semitone. Yet the major semitone and the minor semitone together do precisely make up a tone. So if the major semitone were to be the exact half of a tone, it would follow — since every tone is made up from two of its halves — that the minor semitone would be half of some other tone, which is not true, as has already been shown.

In view of the things we have said about the minor semitone and the comma, however, the major semitone has been added before those numbers, [and] you shall inquire very well for yourself. But how the minor semitone is disposed among the keys of the hand, I shall treat in b-fa-♭-mi.

The tone contains more than eight commas, but fewer than nine, as was easily shown by Boethius. Let A be the number 262,144, and B the number that is a distance of five tones away from A, namely 472,392. Let C, however, be the double number of A, which is the octave, namely 524,288, but D the number that is removed from A by the span of six tones, [namely] 531,411. So D is only a comma away from the number C, [namely,] the difference between six tones and octave. And let this difference, namely, the amount of D in excess of C, be E, namely 7,153.

Distat autem D a numero B prescise spatio toni, quanto distant sex toni a quinque: que differentia, id est excessus ipsius D ad B, sit F, utputa 59049.

His itaque taliter dispositis arguitur in hunc modum: Si numerum E novies auxero, procreabo numerum 64377, qui sit G. Si vero octies multiplicem, eveniet numerus 57224, qui sit H. Sed constat, quod G numerus maior est quam F. Constat etiam F numerum maiorem esse quam H. Et est F numerus toni differentia, G vero est comma novies multiplicatum, H vero octies. Demonstratum est igitur, quod tonus maior est octo comatibus, sed novem comatibus minor.

Attenta predicta demonstratione potes per te reperire, quod semitonium minus plus continet quam tria comata, minus vero quam quatuor, et quot scismata in tono contenta sunt, et multa huiusmodi, que industrie committo tue studiositatis.

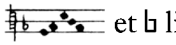
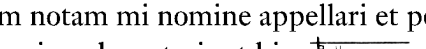
Now D is removed from B by a span of exactly a tone, the amount between six tones and five. And let this difference, that is, the amount of D in excess of B, be F, that is, 59,049.

Having set up these things in this way, let us argue as follows. If I increase the number E by ninefold, I shall make the number 64,377, which is G. But if I multiply it eight times, then I obtain the number 57,224, which is H. Now it is evident that the number G is larger than F. And it is also evident that the number F is larger than H. And F is the number representing the difference of a tone, but G is the comma multiplied nine times, and H eight times. Thus it has been demonstrated that the tone is larger than eight commas, but less than nine commas.

After carefully considering the aforesaid demonstration you can find out for yourself that the minor semitone contains more than three commas but fewer than four, and also how many schismata are contained in the tone, and many things like these, which I leave to the diligence of your studies.

<TERTIA PARS>

B-fa-♭-mi decima clavis est in manu; cui clavi pretextu subtilis eiusdem inventionis potius quam ceteris insistere propono.

Componitur autem ex notis duabus fa et mi, et ex totidem litteris clavem ipsam representantibus eodem nomine vocatis propter eius clavis ydemptitatem, diversimode tamen figuratis naturam diversam ipsius clavis denotantibus; continet namque b litteram, quam b-fa communiter vocamus, que notam sequentem ipsam in hac clave denotat nomine fa appellari ac per semitonium minus super a-la-mi-re decantari, ut hic  et ♭ litteram aliam, quam ♭-mi dicimus, ad denotandum consimilem notam mi nomine appellari et per tonum integrum super a-la-mi-re decantari, ut hic  .

In predictarum litterarum nominibus superstitiosum est morari. Vocant tamen alii b-fa litteram 'b rotundum' et ♭-mi, ♭quadratum', dantes eis nomina secundum formas figurarum

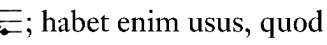
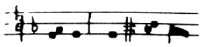
<THIRD PART.>

B-fa-♭-mi is the tenth key in the hand. I propose to dwell on this key more than on the others, because of its subtle invention.

It is put together from two notes, fa and mi, and from as many letters representing this key, [letters that are] called by the same name because of the identity of this key, but figured in different ways to denote the changing nature of this key. For it contains the letter b which we commonly call b-fa, which indicates that the note following [the letter] in this key is to be called by the name fa, and to by sung on the minor semitone above a-la-mi-re, as here: . And the other letter [is] ♭ which we call ♭-mi, to indicate that this same note is to be called by the name mi and to be sung a whole tone above a-la-mi-re, as here: .

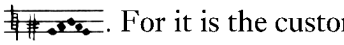
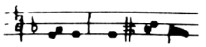
Yet it is overzealous to dwell on the names of the aforesaid letters. For others call the letter b-fa "round b," and ♭-mi "square ♭," giving them names according to the forms of the

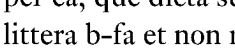
ruditer speculantes in hac arte. Alii subtilius b-fa litteram 'b molle' vocant et ♮-mi, ♭ durum', eo quod cantus, qui sit per fa in hac clave, dum ab f-fa-ut ad ipsam ascendimus, mollius suaviusque se prebet auribus quam ille, qui per mi cantatur. Non enim tantum dulcedinis inest tritono quantum proportioni sesquitertie, que dyatesseron nuncupatur.

Sed quia tritonus, quamvis durior sit, non debeat omnino reici ut contemptus, inventa fuit hec clavis, in quam clavem tam dyatesseron quam tritonus, diversis tamen vicibus, possint ter<m>i<n>ari. Quod potius antiqui voluerunt in hac clave quam in alia, quia mediat quodammodo manum, cum usque ipsam inclusive manus procedit per tria tetracorda coniuncta, id est per ter dyatesseron semitonio super dytonum collocato, quibus totidem tetracorda alia adiunguntur cum tono, disiuncta tamen a primis spatio semitonii maioris, dytono super semitonium minus collocato; que quidem disiunctio fieri habet in hac clave. Idem est in decima septima clavis que ad hanc dyapason resonat et idem nomen est sortita. Advertendum tamen est, quod hec clavis indifferens est ad utrumlibet, puta ad fa et ad mi, si in ea neutra litterarum scribetur, dumtamen vicine claves cognoscantur, licet secundum usum in hoc casu prestantior sit ad mi, ut hic  ; habet enim usus, quod in ea non cantetur fa nisi preposita littera b-fa; et est forte ratio, quare prestantior sit ad mi, ut ipsa decima clavis dyapason consonet super secundam clavem manualem, puta ♮-mi, nec est necesse secundum usum, ut umquam in ea scribatur littera ♮-mi, dumtamen clavis cognita sit, nisi dum a cantu b mollis transire volumus ad cantum ♭ duri, ut hic  , quo casu secunda littera non extorquet sed retorquet extinguendo effectum littere prime. Cognitam ipsam clavem b-fa-♮-mi dico, dum clavis alia in eadem dispositione sit nota, ut in predicto exemplo clavis c-sol-fa-ut est nota per litteram c positam in linea, ergo et b-fa-♮-mi et

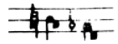
figures, thus speculating crudely in this art. Others, more subtly, call the letter **b-fa** "soft b" and **♮-mi** "hard ♮," because song which has **fa** in this key sounds softer and more sweetly to the ears, when we ascend to it from **f-fa-ut**, than the one that is sung by **mi**. For there is not so much sweetness in the tritone as [there is] in the sesquitertia proportion, which is called the fourth.

Yet although the tritone is harder, it must not be altogether rejected as though held in contempt. Therefore a key was invented on which both the fourth and the tritone could end, albeit at different times. And the ancients wanted [that alternative] on this key rather than on another, for it mediates the hand in some way. For up to and including this [key], the hand proceeds by three conjunct tetrachords, that is, three [intervals of] a fourth positioned on the semitone above the third. To these are joined the same number of tetrachords with a tone [that is, tritones], but separated from the first [tetrachords] by the space of a major semitone [between b♭ and b♮], with a third [c–e] being placed above the minor semitone [b♮–c]. And such disjunction must happen only on this key. The same thing happens on the seventeenth key, which is resonant with this octave and has been given the same name.

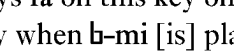
Yet one should note well that if neither of the letters [**b** or **♮**] is written with it, this key is indifferent with regard to either [note], that is, **fa** and **mi**, provided that the neighboring keys can be recognized. Yet according to custom, it would be more excellent in this case, if it were on **mi**, as here:  . For it is the custom that **fa** is not sung on [this key] unless the letter **b-fa** is placed before it. And the reason that it would be more excellent on **mi**, perhaps, is that this tenth key would sound the octave above the [third] key of the hand, that is, [**B-mi**]. Nor is it necessary, in regular custom, that one should ever write the letter **♮-mi** so long as the key is known, except when we wish to move from song [in] soft **b** to song [in] hard **♮**, as here:  . In this case the second letter would not turn away [from the key] but turn back, cancelling out the effect of the first letter. I say that this first key **b-fa-♮-mi** is known when another key is marked in the same disposition, like the key **c-sol-fa-ut** in the aforesaid example is marked by the letter **c** notated on a line. Therefore **b-fa-♮-mi**

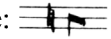
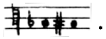
relique sunt cognite per ordinem clavium manualium, et equebene sicut dicta littera c dat cognoscere c-sol-fa-ut, daret a littera in linea cognoscere a-la-mi-re et reliqua; nec est necesse unam ponere plus quam aliam, ut dixi in principio huius libelli. In hac clave, licet mixta sit, non fiunt mutationes ut in aliis. nam per ea, que dicta sunt, in hac clave non dicitur fa nisi prepositа littera b-fa et non mi nisi prepositа ♭-mi, quando semel littera b-fa precesserit: si ergo in hac clave fa mutaretur in mi, sequeretur contradictio, quod littera b-fa preponeretur et non postponeretur vel quod utraque, tam b-fa quam ♭-mi, littera preponeretur et sic una et eadem nota graviter et acute pronuntiaretur respectu suiipsius, quod absurdum est, quamvis hoc alias in quodam contratenore ponere fuerim conatus, ut hic  , cuius note primam medietatem per b-fa et secundam medietatem per ♭-mi nitebar ululare.

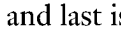
Discrepat iterum hec clavis ab aliis, nam cum inter mi at fa regulariter est semitonium minus, ut hic  . inter tamen mi et fa in hac clave est semitonium maius, ut hic  . Quare autem ibi fit semitonium maius, ratio demonstrat, quoniam prima illarum non distat a clave c-sol-fa-ut nisi spatio semitonii minoris, secunda vero spatio integri toni; sed constat tonum componi ex semitoniis maiore et minore prescise, igitur secunda gravior est prima in quantitate predicta, puta semitonii maioris.

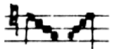
Ex quo infero duo correlaria: Primum quod due note diversarum clavium minus a se distant, id est minorem proportionem faciunt, quam due predicte in eadem clave situate. Secundum correlarium est hoc, quod cantus cromaticus inter claves nostras leviter figuratur, ut ecce  , ubi tetracordum, id est dyatesseron, que est inter primam et ultimam, dividitur in lymma, apothomem et triemitonium; aliter tamen disponitur cantus huiusmodi inter cordas et aliter inter claves; et dicitur a 'croma', quod est 'color', quia in auditu colorem sapit, non tamen tantum

and the others are known from the order of keys in the hand. Just as the said letter **c** allows us to know **c-sol-fa-ut**, it allows us to tell **a-la-re-mi** and others from the letter on the line. Nor do we need to write one more than the other, as said at the beginning of this booklet.

Although this key is composite, there are no mutations as there are in others. For according to what we have said, one says **fa** on this key only when the letter **b-fa** is placed before it, and **mi** only when **♭-mi** [is] placed before it in the event that the letter **b-fa** had already once preceded it. For if **fa** in this key were to be mutated into **mi**, there would arise a contradiction, [namely,] that the letter **b-fa** would have been placed before and not after, or that both letters, **b-fa** and **b-mi**, would be placed before, and thus one and the same note would have to be sung both low and high with respect to itself, which is absurd, even though I have attempted to notate this elsewhere in some contratenor, as here:  , the first half of which note I have tried to howl on **b-fa** and the second half on **b-mi**.

This key is different from the others again in that normally there is a minor semitone between **mi** and **fa**, as here:  , but in this key there is a major semitone between **mi** and **fa**, as here:  . Yet reason demonstrates why there is a major semitone in that place, for the first of those [b♮] is removed from the key **c-sol-fa-ut** by the span of a minor semitone, but the second [b♭] by the span of a whole tone. Yet it is clear that the tone is made up exactly from a major semitone and a minor, therefore the second [b♭] is lower than the first [b♮] by the aforesaid amount, that is, a major semitone.

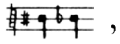
There are two corollaries which I deduce from this. The first is that two notes of different keys are less far apart, that is, have a smaller proportion, than the aforesaid two which are situated within the same key. The second corollary is that chromatic song is easily figured among our keys, as you see here:  , where the tetrachord, that is, the fourth, between the first and last is divided into a limma, an apothome, and a tri-hemitone. For such song is disposed in one way between strings and in another way between keys. And it is named after *chroma*, which is color, because one tastes a color in the hearing, but not so much as the diatonic, as

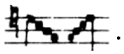
quantum dyatonicus, ut hic  .

Enarmonicus vero cantus ita disponi non potest, quia constat ex dyesi et dyesi et dytono; dyesis enim, quam dyascisma, id est medietatem minoris semitonii, Boetius appellat, nullatenus inter claves nostras, ut hic ex sequentibus patebit, potest consignari nec dytonus inter duas, et quamvis enarmonicum cantum Boetius aptissime dicat coaptatum, forsitan yronice locutus est, ut dicit commentator eius Linconensis.

Ex predictis aliqualiter incipit elucescere natura litterarum clavis presentis, videlicet quod b-fa littera signum est depressionis note sequentis in eadem clave et ♭-mi nota elevationis. Patet etiam directa contrarietas in dictis litteris, quoniam quantum b-fa deprimit, tantum ♭-mi acuit, quia utraque per semitonium maius precise.

Potest ergo pro principio supponi in hac arte regula sequens, quod b-fa littera gravat, ♯-mi littera acuit quelibet notam sue clavis extra naturam in quantitate semitonii maioris, quod intelligendum est, dum nulla littera precesserit, nam tunc non faceret quam extinguere naturam littere precedentis; fuit ergo, ut apparet, semitonium maius principalius repertum in hac clave.

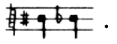
Secundo suppono, quamvis hec due littere magis proprie locum optinent in hac clave, quod nichilominus licitum est ipsas statui in aliis, ymo in G-ut et alibi secundum exigentiam ipsius cantus, prout in pluribus vidi motetis autenticis; sed tunc non erunt representative clavium, sed signum gravitatis et acutiei notarum sequentium in illa clave, in qua huiusmodi littere b-fa et ♭-mi posite sunt, quas quidem notas tunc proprie extorquent extra naturas suas proprias in quantitate sepe dicta. Et quantum una littera notam extorquet et distemperat, tantum sequens littera temperat et retorquet et per consequens reducit ad suam propriam mansionem, ut hic  , in quo exemplo secunda equebene velut prima superaret veram suam clavem, puta c-sol-fa-ut, nisi per b-fa litteram ad propriam naturam

here:  .

Yet enharmonic song cannot be disposed in this way, for it consists of a diesis, a diesis, and a major third. And the diesis, which Boethius calls the diaschisma, that is, half of the minor semitone, cannot be designated in any way among our keys, as shall be apparent from the things that follow, nor the major third between two [diaschismata]. And although Boethius calls enharmonic song "most suitably adapted," perhaps he spoke ironically, as his Linconian commentator says.

From the aforesaid things, the nature of the letters in the present key begins to emerge more clearly, namely, that the letter **b-fa** is a sign of the lowering of the subsequent note in the same key, and **♭-mi** a mark of raising. There is also a direct contrariety apparent in the said letters, for **b-fa** lowers as much as **♭-mi** raises, since both [do so] exactly by the major semitone.

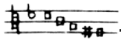
But as a principle in this art we may posit the following rule, that the letter **b-fa** lowers and the letter **♯-mi** sharpens every note by the quantity of a major semitone beyond the nature of its key. But [this rule] is to be understood only [in situations] where no [other] letter has preceded. Otherwise [neither letter] would not do anything except cancel out the nature of the preceding letter. So the major semitone, it appears, was found more principally in this key.

Secondly, although these two letters more properly find their place in this key, I do suppose that it is nevertheless permissible for them to be placed in other [locations], even in **G-ut** and elsewhere, according to what the song demands, as I have seen in many motets authored [?by myself]. But in that case they shall not represent keys, but [rather they shall be] signs of the lowering and raising of the notes that follow in the key before which those letters **b-fa** and **♭-mi** are placed. Which notes are then properly turned away from their proper natures, by the quantity already mentioned many times. And just as much as a letter turns and tempers a note, so much does the next letter temper and turn and consequently restore it to its proper home, as here:  . If the second [note], in this example, had not been restored to its proper nature by the letter **b-fa**, it would have remained above its proper key, that is, [above] **c-sol-fa-ut**, just

reverteretur; et est inter ipsas semitonium maius. Ratio quare in aliis clavibus equebene licet uti talibus litteris est, quia propter cantum figure invente sunt et ergo cantui subordinate et non econtra.

Tertio suppono, quod quelibet due claves proxime regulariter, prout manus confecta est, statuent tonum vel semitonium minus, ut patet discurrendo per omnes viginti claves a G-ut usque ad e-la, et loquor quando stant in suis propriis naturis.

Hiis suppositis sequuntur multe conclusiones pulchre, quarum aliquas, que viam dabunt ad nondum inventas, quantum est de presenti explanabo.

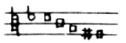
Prima conclusio: Duo vel tria minora semitonia possibile est disponi, ut hic  , inter quas sine littera ♭-mi esset tonus cum semitonio minore, et cum tonus componitur ex maiore et minore, essent ibi duo minora et unum maius; sed per secundam suppositionem licitum est undique poni litteram ♭-mi, igitur per primam suppositionem acuitur secunda extra naturam suam per semitonium maius, igitur per tantumdem distantia illa est restricta: remanebunt ergo prescise duo minora. Exemplum de tribus ut hic  .

Inter clavem c-sol-fa-ut et ultimam notam sunt duo minora, ut iam probatum est; sed nota prima excedit clavem c-sol-fa-ut solum in minore semitonio per litteram b-fa, que a tono, qui est inter d-la-sol-re et c-sol-fa-ut, resecat semitonium maius per primam suppositionem; manent igitur ibi tria semitonia tunc minora. Potest ergo tradi practica huius conclusionis sub hiis regulis: Si distantia semiditonalis restringitur per alteram litterarum ipsius b-fa-♭-mi, exibunt duo minora; si dyatesseron restringitur per ambas, tria minora semitonia resultabunt. Hanc conclusionem ponere me monent aliqui, qui dicunt duo vel tria semitonia minora non posse decantari.

as much as the first. And there is a major semitone between those [notes]. The reason why these letters may be used equally well in other keys is that figures have been invented for the sake of song, and therefore are subservient to song, not the other way round.

Thirdly, I do suppose that any two neighboring keys shall state the tone or minor semitone in regular fashion, according to the way the hand is put together, as is apparent when one runs through all twenty keys from **G-ut** up to **e-la**. And I am speaking of when they are standing in their proper natures.

Having supposed these things, there follow many beautiful conclusions, among which there are some that shall point the way to things not yet invented, which I shall explain as much [as is possible] at the present time.

First conclusion: It is possible for two or three minor semitones to be disposed like this:  . Without the letter **♭-mi** there would be a tone with minor semitone between [these notes]. And since the tone is put together from the major and minor [semitone], there would be two minor and one major there. Yet according to the second supposition it is permissible to place the letter **♭-mi** anywhere. According to the first presupposition, therefore, the second [note] is raised beyond its nature by a major semitone. And therefore it is restricted by the same distance: there shall therefore remain exactly two minor [semitones]. Example of three, like this:  .

Between the key **c-sol-fa-ut** and the last note there are two minor [semitones], as has already been demonstrated. But the first note exceeds the key **c-sol-fa-ut** only by a minor semitone, because of the letter **b-fa** which according to the first presupposition cuts a major semitone from the tone between **d-la-sol-re** and **c-sol-fa-ut**. There remain, therefore, three minor semitones. The practice of this conclusion can be conveyed in these rules. If the distance of the major third is restricted by another of the letters of this **b-fa-♭-mi**, then two minor [semitones] shall result; if the fourth is restricted by both, then three minor semitones shall result. I have been reminded to posit this conclusion because of some who assert that it is not possible to sing two or three minor semitones.